Parallel and Perpendicular Lines

Parallel Lines

The slopes of parallel lines are always equal.

- Example:
  - The equations \( y = 4x + 1 \) and \( y = 4x + 3 \) are parallel, because they both have a slope of 4.

Perpendicular Lines

The slopes of perpendicular lines are always opposite reciprocals of each other.

- Example:
  - The equations \( y = \frac{2}{3}x + 2 \) and \( y = -\frac{3}{2}x + 5 \) are perpendicular because their slopes are opposite reciprocals of each other.

  - The term opposite reciprocal simply means to flip the slope and change the sign.
    - For example, if \( m = \frac{4}{5} \), then a slope perpendicular to that would be \( m = -\frac{5}{4} \).

Two slopes that are perpendicular will always have a product of \(-1\).

- Example:
  - \( \frac{4}{5} \cdot -\frac{5}{4} = -\frac{20}{20} = -1 \).
Writing Equations of Parallel and Perpendicular Lines

There are 3 steps to writing equations of parallel and perpendicular lines.

1. Determine the slope of the new line.
2. Pick which equation to use:
   a. Slope-intercept: \( y = mx + b \)
   b. Point-Slope: \( y - y_1 = m(x - x_1) \)
3. Substitute values.

Ex 1 – Write the equation of a line perpendicular to \( y = -\frac{1}{3}x - 5 \) through the point (-1, 4). Write the final answer in slope-intercept form.

- Step 1: Determine the slope of the new line.
  - Since it must be perpendicular to the given line, the slope of our new line must be \( m = 3 \).

- Step 2: Pick equation.
  - Since we have the point (-1, 4), we should use the point-slope equation of a line: \( y - y_1 = m(x - x_1) \).

- Step 3: Substitute values.
  - Since \( m = 3 \) and \((x_1, y_1) = (-1,4)\), we can substitute those into the point-slope equation:
    \[
    y - y_1 = m(x - x_1) \\
    y - 4 = 3(x - (-1)) \\
    y - 4 = 3(x + 1).
    \]

Now the problem says to write the final answer in slope-intercept form. To do this, we simply solve for \( y \).

\[
\begin{align*}
y - 4 & = 3(x + 1) \\
y - 4 & = 3x + 3 \\
y & = 3x + 7
\end{align*}
\]

So the line parallel to \( y = -\frac{1}{3}x - 5 \) through the point (-1, 4) is \( y = 3x + 7 \).